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Research Article

A New Robust Estimation Approach to Dual Response Surface Models

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ABSTRACT

Robust parameter design is one of the methods used to determine the optimum operating conditions of systems. In robust parameter design, dual response surface methodology plays a crucial role in identifying optimal settings that minimize variability while maintaining desired performance. The methods generally used in dual response surface models are based on the assumption of normal distribution. Nonetheless, using techniques that rely on traditional normality assumptions to model non-normally distributed data often leads to the failure of quality improvement processes. In cases where there is deviation from normality, using robust estimators in response surface models for mean and variance has become very popular in recent years. This study focuses on skewed data, and thanks to the proposed approach, it is aimed to produce more robust results than the models in the literature by using the BS82 robust M estimator instead of standard deviation, with modelling for confidence intervals. The main advantage of the proposed approach is that it provides a robust solution that considers the skewed nature of the experimental data distribution. To demonstrate the validity of the approach, the newly developed model has been examined *printing process* data is frequently discussed in the literature.

Keywords: *Dual response surface, robust estimation, BS82, confidence interval*

İkili Yanıt Yüzey Modellerine Yeni Bir Dayanıklı Tahmin Yaklaşımı

ÖZ

Dayanıklı parametre tasarımı, sistemlerin optimum çalışma koşullarını belirlemek için kullanılan yöntemlerden biridir. Dayanıklı parametre tasarımında, ikili yanıt yüzey metodolojisi, değişkenliği en aza indirirken istenen performansı koruyan optimal ayarları belirlemede önemli bir rol oynar. İkili yanıt yüzeyi modellerinde genellikle kullanılan yöntemler, normal dağılım varsayımına dayanmaktadır. Ancak normal dağılmayan bir veriyi modellemek için geleneksel normallik varsayımlarına dayanan teknikleri kullanmak genellikle kalite iyileştirme süreçlerinin başarısız olmasına neden olur. Normallikten sapma olduğu durumlarda, ortalama ve varyans için yanıt yüzey modellerinde dayanıklı tahmin edicilerin kullanılması son yıllarda oldukça popüler hale gelmiştir. Bu çalışma çarpık veriler üzerinde yoğunlaşmış olup, önerilen yaklaşım sayesinde, güven aralığına dayalı yanıt yüzey modellemede standart sapma yerine BS82 dayanıklı M tahmin edicisi kullanılarak literatürdeki modellerden daha dayanıklı sonuçlar üretilmesi hedeflenmiştir. Önerilen bu yaklaşımın temel avantajı, deneysel veri dağılımının çarpık yapısını dikkate alan daha dayanıklı bir çözüm sunmasıdır. Yaklaşımın geçerliliğini ortaya koymak için, yeni elde edilen model literatürde sıkça tartışılan *baskı süreci* verisi üzerinde incelenmiştir.

Anahtar Kelimeler: *İkili yanıt yüzey, dayanıklı tahmin, BS82, güven aralığı*

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I. INTRODUCTION

This study introduces a novel approach to enhance the robustness of parameter design in systems where data distribution deviates from normality. Taguchi introduced robust parameter design to make the system as robust as possible against undesirable fluctuations in process performance [1]. Robust parameter design and the Taguchi philosophy have attracted great attention in different industrial fields. However, there are some criticisms of Taguchi's experimental methods and analysis techniques. Many researchers, such as Box and Vining & Myers, have proposed new methodologies based on these criticisms [2, 3]. Vining and Myers developed a new procedure for off-line quality improvement approaches by combining some effective features of the response surface methodology and the Taguchi method revealed [3]. The response surface methodology, first introduced by Box and Wilson, is an effective procedure for modelling the possible process relationship between quality characteristics and design factors to determine optimum operating conditions [4]. Vining and Myers introduced dual response surface models based on obtaining separate response surfaces of mean and variance [3]. Thus, the dual response surface methodology achieves the primary goal of robust parameter design by optimizing the primary response depending on the secondary response value. This new approach to response surface methodology has attracted a lot of attention because it produces robust results and has been the subject of many studies, forming the basis for studies on this subject. Del Castillo and Montgomery suggested using standard nonlinear programming techniques [5]. Lin and Tu suggested minimizing the mean square error (MSE) criterion for the dual response surface problem by focusing on process bias as well as variability [6]. Copeland and Nelson proposed a new model by working on the mean square error criterion [7].

The robust parameter design methodology seeks to identify the optimal levels of design factors by fine-tuning the model developed from experimental data. Nevertheless, the limited nature of the experimental data means that accurately estimating the distribution and model parameters can be challenging, often resulting in estimation errors due to inadequate data or unpredictable random effects. Elsayed and Chen demonstrated that these errors in parameter estimation can cause the derived model to deviate significantly from the true model, potentially leading to suboptimal operating conditions that are not aligned with the actual optimal values [7, 8]. Overlooking these uncertainties, where the objective function is typically optimized based on point estimates, may result in a misleading fit and substantial variances in system performance [9]. Park and Cho [10] presented a study for cases where the distribution of responses is not normal. When normality assumptions are not met, using traditional mean and standard deviation increases the amount of bias in the estimates. They suggested using robust estimators in such situations. Lee et al. [11] mentioned robust modelling for normal and skewed data. Modelling obtained under unrealistic assumptions about data distribution can lead to incorrect decisions for practitioners.

The probability distribution of experimental data is crucial in robust design. Typically, response surface modeling is based on the assumption that the data follows a normal distribution. However, this assumption may not hold in real-world industrial scenarios. Responses may follow distributions such as Poisson, Exponential/Gamma, or Bernoulli in various engineering contexts. As a result, because process mean and variance are highly dependent on the underlying data distribution, relying on traditional normality assumptions to model non-normal responses can often lead to ineffective quality improvement techniques.

In recent years, the use of robust estimators in mean and variance response surface models has become very popular in cases where there is deviation from normality. Boylan and Cho investigated the performance of robust estimators, highlighting their greater efficiency and resilience when dealing with outliers [14]. In recent years, several methods utilizing confidence regions have been proposed to mitigate the effects of uncertainties in model parameters on robust parameter design. Ouyang et al. [15], Zeybek [12], and Zeybek [13] have presented new modelling approaches based on confidence intervals. The rest of this study is organized into three sections: The following section introduces the proposed

approach. Next, a popular experimental design study is used to demonstrate the validity of this approach. Finally, the study ends with a conclusion.

II. THE PROPOSED APPROACH

In the dual response surface problem, the use of sample mean and variance point estimators was suggested by Vining and Myers [3]. The quality characteristic is defined as y, where k controllable factors are $x_1, x_2,\ldots, x_k$ and each experiment is repeated r times at n-trial points, y_{ij} is ith at trial point, jth response.

In the case where it is expressed as a response ith point estimators of the mean and standard deviation of the relevant quality characteristic at the trial point, for $j = 1, \ldots, r$ and $i = 1, \ldots, n$

$$\overline{y_i} = \frac{1}{r}\sum_{j=1}^{r} y_{ij}$$

and

$$s_i = \sqrt{\frac{1}{r-1}\sum_{j=1}^{r}\left(y_{ij} - \overline{y_i}\right)^2}$$

"Mean" and "Standard Deviation" response surface models are also created as given below:

$$\hat{\mu}(x) = \hat{\gamma}_0 + \sum_{i=1}^{k}\hat{\gamma}_i x_i + \sum_{i=1}^{k}\hat{\gamma}_{ii} {x_i}^2 + \sum_{i<t}^{k}\sum \hat{\gamma}_{it} x_i x_t$$

$$\hat{\sigma}(x) = \hat{\delta}_0 + \sum_{i=1}^{k}\hat{\delta}_i x_i + \sum_{i=1}^{k}\hat{\delta}_{ii} {x_i}^2 + \sum_{i<t}^{k}\sum \hat{\delta}_{it} x_i x_t$$

where $\hat{\gamma} = (X'X)^{-1}X'w_\mu$ and $\hat{\delta} = (X'X)^{-1}X'w_\sigma$.

$w_\mu = (\overline{y_1}, \overline{y_2}, \ldots, \overline{y_n})'$ and $w_\sigma = (s_1, s_2, \ldots, s_n)'$ are the mean and standard deviation vectors. X shows the design matrix.

Equation (1) has been suggested to use $(1-\alpha)$ confidence interval instead of the process average in non-normal processes as follows: [16].

$$\left(\overline{y_i} + \frac{\hat{\mu}_{3,i}}{6r\, s_i^2}\right) \mp t_{\frac{\alpha}{2},r-1}\frac{s_i}{\sqrt{r}} \qquad (1)$$

where $t_{\frac{\alpha}{2},r-1}$, is the t-distribution with $r-1$ degrees of freedom for %(1-α)×100 confidence level. $\hat{\mu}_{3,i}$ is the third central moment estimated from the sample and is used to define skewness. This approach adjusts the discrepancy between the median and the mean caused by the skewness of the distribution, ensuring that the median remains an unbiased estimator of the mean. Additionally, the midpoint between the lower endpoint (LE) and upper endpoint (UE) response surfaces is utilized as a measure of location performance. The LE and UE for the interval provided in Equation (1) at the ith design point are calculated as given below:

$$LE_i = \left(\overline{y_i} + \frac{\hat{\mu}_{3,i}}{6r\, s_i^2}\right) - t_{\frac{\alpha}{2},r-1}\frac{s_i}{\sqrt{r}} \tag{2}$$

$$UE_i = \left(\overline{y_i} + \frac{\hat{\mu}_{3,i}}{6r\, s_i^2}\right) + t_{\frac{\alpha}{2},r-1}\frac{s_i}{\sqrt{r}} \tag{3}$$

The resulting LE and UE response surface functions are therefore proposed as follows:

$$\widehat{LE}(x) = \hat{\alpha}_0 + \sum_{i=1}^{k}\hat{\alpha}_i x_i + \sum_{i=1}^{k}\hat{\alpha}_{ii}{x_i}^2 + \sum_{i<t}^{k}\sum\hat{\alpha}_{it}x_i x_t \tag{4}$$

$$\widehat{UE}(x) = \hat{\beta}_0 + \sum_{i=1}^{k}\hat{\beta}_i x_i + \sum_{i=1}^{k}\hat{\beta}_{ii}{x_i}^2 + \sum_{i<t}^{k}\sum\hat{\beta}_{it}x_i x_t \tag{5}$$

where $\hat{\alpha} = (X'X)^{-1}X'w_{LE}$, $\hat{\beta} = (X'X)^{-1}X'w_{UE}$; $w_{LE} = (LE_1, LE_2, \dots, LE_n)'$ and $w_{UE} = (UE_1, UE_2, \dots, UE_n)'$.

As outlined in the theory of interval analysis—refer to [15, 17, 18]—the midpoint of an interval can serve as an indicator of location performance. The response surface function for the estimated position performance is expressed as in the following equation:

$$\hat{\mu}(x) = \frac{\widehat{UE}(x) + \widehat{LE}(x)}{2} \tag{6}$$

Traditionally, the sample standard deviation is used to obtain the standard deviation's response surface. In this study, a robust estimator, BS82, is recommended as an alternative for standard deviation in response modelling.

Robust estimators were first introduced by Tukey, who based his work on the robustness of some simple substitutes [19]. Subsequently, Huber's work considering the class of neighboring alternatives attracted considerable attention in the literature [20]. The second basic approach is to derive robust confidence intervals from robust tests proposed by Huber [21, 22]. A large number of estimators, called M-estimators, have been developed that are effective and robust to many such situations [23-25]. As one of the most effective and robust estimators in Gross's study, BS82 is expressed as follows:

$$\hat{\sigma}_{BS82} = (8.2S_0)\sqrt{n\left(\frac{\sum\psi^2(z_i)}{\sum(\psi'(z_i))^2}\right)} \tag{7}$$

where $T_0 = median(y_i)$, $S_0 = median|y_i - T_0|$, $|z_i| = |(y_i - T_0)/(8.2S_0)|$,

$$\psi(z) = \begin{cases} z(1-z^2)^2 & , \ if \ |z| \le 1 \\ 0 & , \ if \ |z| > 1 \end{cases}.$$

This study considers the BS82 M-estimator to obtain robust response estimates. In the optimization phase, the objective function and constraints are as given below:

For the larger-the-better (LTB) case: $Max\ \widehat{UE}(x)$ $s.t.\ \ \hat{\sigma}(x) \leq \sigma_0$ and $x^* \in R$

For the nominal the best (NTB) case: $Min\ \ \hat{\sigma}(x)$ $s.t.\ \ \hat{\mu}(x) = \tau$ and $x^* \in R$

where τ denotes the pre-determined process target. The experimental region can be described as $-1 \leq x_i \leq 1,\ i = 1, \dots, k$ for cuboidal designs $x'x \leq \rho^2$ for spherical designs, where ρ represents the design radius. For the NTB scenario, if relaxing the zero-bias assumption is intended to improve process variance estimation, then the squared-loss optimization model, Min $MSE = (\hat{\mu}(x) - \tau)^2 + \hat{\sigma}(x)^2$, can be employed during the optimization phase.

III. APPLICATION EXAMPLE

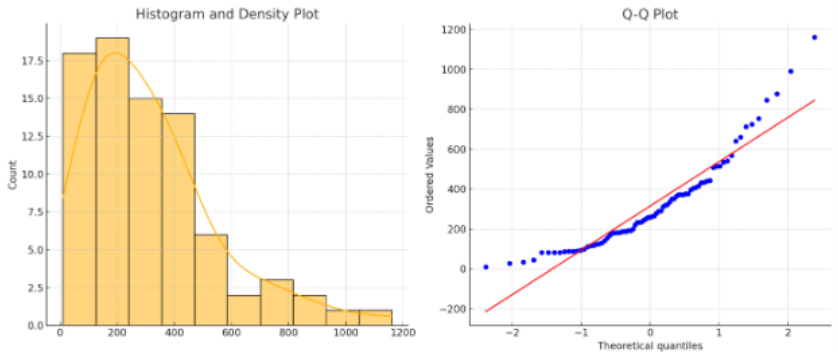
This part of the study demonstrates the application of the proposed interval robust design method on a printing process case study. As detailed in Table 1, 3x3x3 factorial design with three replications was conducted to evaluate the impact of speed (x_1), pressure (x_2), and distance (x_3) on a printing machine's ability (y) to apply colored inks to package labels. The printing process requires a target of 500 and a desired standard deviation of 60.

Table 1. *The printing process study with* $\hat{\sigma}_{BS82}$.

i	x_1	x_2	x_3	y_1	y_2	y_3	LE	UE	$\hat{\sigma}_{BS82}$
1	-1	-1	-1	34	10	28	-7.23	54.83	11.59
2	0	-1	-1	115	116	130	99.67	141.34	0.88
3	1	-1	-1	192	186	263	108.16	320.96	5.28
4	-1	0	-1	82	88	88	77.32	94.53	0.02
5	0	0	-1	44	178	188	-68.32	344.77	0.02
6	1	0	-1	322	350	350	300.16	380.48	0.02
7	-1	1	-1	141	110	86	43.96	180.97	23.53
8	0	1	-1	259	251	259	244.76	267.71	0.02
9	1	1	-1	290	280	245	212.56	329.96	22.11
10	-1	-1	0	81	81	81	80.98	81.02	0.01
11	0	-1	0	90	122	93	58.13	145.94	2.64
12	1	-1	0	319	376	376	274.54	438.05	0.02
13	-1	0	0	180	180	154	133.72	208.31	0.02
14	0	0	0	372	372	372	371.98	372.02	0.01
15	1	0	0	541	568	396	270.07	729.68	74.42
16	-1	1	0	288	192	312	105.11	420.61	59.21

17	0	1	0	432	336	513	206.60	646.85	75.17
18	1	1	0	713	725	754	678.59	783.33	19.14
19	-1	-1	1	364	99	199	-110.62	554.30	116.63
20	0	-1	1	232	221	266	181.77	298.33	21.81
21	1	-1	1	408	415	443	376.32	468.34	17.27
22	-1	0	1	182	233	182	126.48	272.78	0.02
23	0	0	1	507	515	434	373.52	595.30	7.03
24	1	0	1	846	535	640	282.41	1068.50	140.41
25	-1	1	1	236	126	168	39.23	315.04	48.29
26	0	1	1	660	440	403	158.57	848.90	92.93
27	1	1	1	878	991	1161	657.13	1364.94	123.06

To examine the shape of the data distribution, graphical analysis was conducted—refer to Figure 1. The data clearly lacks a symmetric distribution, suggesting that it is not suitable for modeling using normal theory.



(a) ***(b)***

Figure 1. (a) *Histogram and density plot* ***(b)*** *Q-Q plot*

For illustration, a significance level of $\alpha = 0.05$ is selected, and the interval robust design approach is established with a 95% confidence level. The LE, UE and s_{bm} for each design point are determined using Equations (2)-(3) and (7), as shown in Table 1. Subsequently, the fitted response surfaces for LE, UE and the BS82 standard deviation in the printing process data are represented by the following equations:

$$\widehat{LE}(x) = 239.63 + 148.39x_1 + 71.38x_2 + 59.65x_3 + 21.97{x_1}^2 - 18.14{x_2}^2 - 70.20{x_3}^2 + 47.01x_1x_2 + 62.83x_1x_3 + 8.90x_2x_3$$

$$\widehat{UE}(x) = 414.98 + 205.66x_1 + 147.51x_2 + 203.94x_3 + 41.35{x_1}^2 - 26.18{x_2}^2 + 13.91{x_3}^2 + 85.37x_1x_2 + 88.22x_1x_3 + 78.87x_2x_3$$

$$\hat{\sigma}_{BS82}(x) = 8.7 + 7.91x_1 + 15.96x_2 + 28.0x_3 + 14.4{x_1}^2 + 10.9{x_2}^2 + 9.4{x_3}^2 + 11.6x_1x_2 + 10.3x_1x_3 + 6.7x_2x_3$$

The fitted mean response function $\hat{\mu}(x)$ is derived using Equation (6) based on Equations (4) and (5). During the optimization phase, the proposed optimization approach is implemented within the cuboidal region $(-1 \leq x_i \leq 1, i = 1,2,3)$ for both NTB and LTB cases. Tables 2 and 3 provide a comparison between the results of the proposed modeling approach and those obtained using existing methods.

In the study by Zeybek [13], it was found that the achievable maximum process mean value ranged between 484.09 and 865.68, with an estimated mean response of 674.89. Table 2 presents the results for the LTB case under the cuboidal region, where the optimal solution is identified as $x' = (1.000, 0.307, 0.439)$ with a 95% confidence level. This suggests that there is a 95% confidence that the highest achievable process mean value falls within the range of 486.025 to 872.400, with an estimated mean response of 679.212. Notably, the results derived using the BS82 robust estimator demonstrate a more favourable performance compared to the BWMV estimator used in the referenced study.

Table 2. *A comparative study for the printing process data under the LTB case.*

	x^*	$\hat{\mu}(x)$	$\hat{\sigma}(x)$	**95% CI of the mean** $\widehat{LE}(x)$	$\widehat{UE}(x)$
Proposed approach	$(1.000, 0.307, 0.439)$	679.212	60.00	486.025	872.400
Zeybek (2020)	$(1.000, 0.297, 0.426)$	674.89	60.00	484.09	865.68
Vining and Myers (1990)	$(0.772, 0.307, 0.204)$	557.90	60.00	Unknown	Unknown
DelCastillo and Montgomery (1993)	$(1.000, 1.000, -0.281)$	616.70	60.00	Unknown	Unknown

For the NTB scenario, the optimal solution is $x' = (1.000, 0.017, -0.183)$ with a 95% confidence level. The estimated mean response aligns with the target, $\hat{\mu}(x)$=500, with a standard deviation of 24.774. The MSE of the proposed approach is 613.7679, significantly lower than that of competing methods—approximately 77% lower than Vining and Myers (1990) and about 30% lower than Boylan and Cho (2013). In Zeybek's 2020 study, the MSE was found to be 625.05, indicating that the proposed method offers a more competitive MSE value.

Table 3. *A comparative study for the printing process data under the NTB case.*

	x^*	$\hat{\mu}(x)$	$\hat{\sigma}(x)$	**95% CI of the mean** $\widehat{LE}(x)$	$\widehat{UE}(x)$	**MSE**
Proposed approach	$(1.000, 0.017, -0.183)$	500.00	24.774	387.242	612.758	613.7679
Zeybek (2020)	$(1.000, 0.019, -0.184)$	500.00	25.004	387.25	612.76	625.05
Vining and Myers (1990)	$(0.614, 0.228, 0.1000)$	500.00	51.78	Unknown	Unknown	2680.96
DelCastillo and Montgomery (1993)	$(1.000, 0.118, -0.259)$	500.00	45.10	Unknown	Unknown	2033.74

Tables 2 and 3 effectively demonstrate the potential risks when the underlying data distribution is not considered in response modeling.

IV. CONCLUSION

This paper addresses the challenges posed by skewed experimental data and parameter uncertainties in response models. BS82 is recommended as an alternative robust estimator for modeling process variance, as it tends to deliver better results when normality assumptions are violated. The proposed approach and its effectiveness are demonstrated using a well-known experimental data set. For the NTB case, this interval robust design model achieves the smallest MSE while meeting the target in a skewed printing process data set. Moreover, this approach yields the highest mean for the LTB case within the same data set. The superior performance of the proposed method is primarily due to its ability to leverage the true data distribution's skewness and consider the errors from the fitted model. Traditionally, sample mean and variance estimators fail to account for the skewness in data. The proposed method offers significant insights into the printing process example, especially by adjusting the disparity between the mean and median and by using both resistant and robust estimators for modeling standard deviation, considering the skewed distribution of the experimental data.

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